Role of the gluons in the color screening in a QCD plasma

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The color screening in a QCD plasma, that was studied in a formulation making evident similarities and differences with the electric case, is continued by taking into account the contributions of real gluons. The results, which include a numerical analysis not previously performed, show a damping of the correlation function which, if not exponential, does not differ very much from that form. The role of the temperature, which affect both the population and the dynamics of the quark-gluon system, is found to be relevant.

1. INTRODUCTION

The analysis of the screening in $q\bar{q}$−plasma performed keeping the analogy with the classical treatment of an electric plasma [1] gave as result a slight difference in the screening behavior with distance, with respect to the more standard, exponential decay [2, 3, 4]. Now this analysis is pushed further by introducing the effect of the real gluons which are certainly present in such situations. Once the relevance of the gluons in the quark-quark correlation functions is taken into account one in lead, unavoidably, to consider also the quark-gluon correlations and this is performed in the present paper. While a definite quark population is taken as a initial condition of the problem, the gluon population is considered of thermal origin and is given therefore in terms of a Bose-Einstein density, together with this population also the creation of $q\bar{q}$−pair is considered, with a corresponding Fermi-Dirac density. Being interested in the large distance effects, that part of the interaction of real gluons with quark is considered, which correspond to a t−channel exchange of a virtual gluon, so that it has, suitably redefining color and spin factors, the same form of quark-quark interaction.

The result is a general behavior of the correlation function not very different from that previously found in a pure $q\bar{q}$−plasma, since the analytical form is however awkward a numerical investigation has been performed for both situations without gluons and with gluons. The behavior differs little from the exponential, in both case the possibility of small oscillations is found, but only when the correlation function is already very small.

A discussion of the range of values for which the model can be meaningful and also a comparison with some different treatments and results, in particular with the formulation using the termal Green functions are briefly presented [5].

Three short Appendices contains some details of the needed calculations.

2. STUDY OF THE CORRELATION FUNCTIONS

2.1 General form of the equations

As it has been explained in the Introduction the basic equation for the correlation functions have the same general features as for the pure $q\bar{q}$ populations, but we must distinguish two possible correlation, the two-fermion case and the fermion-boson case. In principle we should look also at the two-boson correlation function, since however our main interest is in the two-fermion correlations while keeping into account the boson-fermion correlation, that enter directly in the equation for the two-fermion function we neglect the effect of the pure bosonic correlation: in so doing we are forced to bound our discussion to situation where the fermion density is dominant, having in mind the nucleus-nucleus collision there are certainly situations, of not too large temperature, where this condition is satisfied.

The ways in which the usual determination of the shielding has been adapted to the case of non commuting charges have been presented in the provios paper and will not be discussed here, only the integrodifferential equation for the two body correlation is reproduced

$$\frac{\partial C_\beta(q_1, q_2)}{\partial r_{1,v}} = -\int_o^\beta d\tau \left[\frac{\partial u(q_1, q_2)}{\partial r_{1,v}} + \right.$$
$$\left. \frac{1}{3V}\sum_{l\neq 1,2}\int d^3 q_l \left[\frac{\partial u(q_1, q_l)}{\partial r_{1,v}} C_{\beta-\tau}(q_l, q_2) + C_\tau(q_l, q_2)\frac{\partial u(q_1, q_l)}{\partial r_{1,v}}\right]\right] \tag{1}$$

where the variables q embody both the space variables r and the color indices and the integration over the auxiliary variable τ takes care of the non commutativity of the charges.

Now a further derivative with respect to r_1 is calculated, then one performs the Fourier transform with respect to the space variables and the Laplace transform with respect to the inverse temperature with the result:

$$-k^2 \check{C}(s;k) = \frac{4\pi\alpha T}{s^2} + \frac{4\pi\alpha}{3Vs}\sum \left[T\check{C}(s;k) + \check{C}(s;k)T\right] \tag{2}$$

The check (as $\check{C}$) means both the Fourier and the Laplace transform; the variable k is the Fourier-conjugated of r and s is the Laplace-conjugated of the inverse temperature β, $4\pi\alpha = g^2$. The factors $\check{C}(s;k)$ and T are matrices so their order is relevant. In order to proceed is is necessary to specify the colour structure of the quarks and of the gluons, so the indices will now must now be displayed and the precise meaning of T and $\check{C}$ is specified. It is not convenient to work with two set of indices, for quark and gluon respectively, so the color charge of the gluon is represented by a traceless tensor with triplet indices, like A_d^c with $A_n^n = 0$.

The term T is specified in this way: the interactions qq and $\bar{q}\bar{q}$ are given by [App.A]

$$I_{a,c}^{b,d} = \frac{1}{2}\left[\delta_a^d \delta_c^b - \frac{1}{3}\delta_a^b \delta_c^d\right]$$

the interaction $q\bar{q}$ is given by $-I$; the interaction qg is given by

$$J_{a,cf}^{b,dg} = \frac{1}{2i}\left[\delta_a^d \delta_c^b \delta_f^g - \delta_a^g \delta_c^d \delta_f^b\right]$$

the interaction $\bar{q}g$ is given by $-J$.

The term $\check{C}$ is indicated with different symbols following its quark and gluon content and is decomposed according to the color structure: for qq and $\bar{q}\bar{q}$ it is called respectively Q and $\bar{Q}$ and decomposed in terms of *triplet* and *sextet* for $q\bar{q}$ it is called M and decomposed into *singlet* and *octet* and for qg or $\bar{q}g$ it is called B or $\bar{B}$ and decomposed into *triplet, sextet* and *15-plet* by means of the following projectors:

$$^3\Pi_{a,c}^{b,d} = \frac{1}{2}\left[\delta_a^b \delta_c^d - \delta_a^d \delta_c^b\right] \qquad\qquad ^6\Pi_{a,c}^{b,d} = \frac{1}{2}\left[\delta_a^b \delta_c^d + \delta_a^d \delta_c^b\right]$$
$$^1\Pi_{a,c}^{b,d} = \frac{1}{3}\delta_a^d \delta_a^d \qquad\qquad\qquad ^8\Pi_{a,c}^{b,d} = \left[\delta_a^b \delta_c^d - \frac{1}{3}\delta_a^d \delta_c^b\right] \tag{3}$$

$$^3P_{a,cf}^{b,dg} = \frac{3}{8}\delta_a^g \delta_c^d \delta_f^b - \frac{1}{8}\left[\delta_a^d \delta_c^g \delta_f^b + \delta_a^g \delta_c^b \delta_f^d\right] + \frac{1}{24}\delta_a^b \delta_c^g \delta_f^d$$
$$^6P_{a,cf}^{b,dg} = \frac{1}{2}\left[\delta_a^b \delta_c^d - \delta_a^d \delta_c^b\right]\delta_f^g - \frac{1}{4}\left[\delta_a^g \delta_c^d \delta_f^b + \delta_a^b \delta_c^g \delta_f^d - \delta_a^g \delta_c^b \delta_f^d - \delta_a^d \delta_c^g \delta_f^b\right]$$
$$^{15}P_{a,cf}^{b,dg} = \frac{1}{2}\left[\delta_a^b \delta_c^d + \delta_a^d \delta_c^b\right]\delta_f^g - \frac{1}{8}\left[\delta_a^g \delta_c^d \delta_f^b + \delta_a^b \delta_c^g \delta_f^d + \delta_a^g \delta_c^b \delta_f^d + \delta_a^d \delta_c^g \delta_f^b\right] \tag{4}$$

The correct normalization of the projector is easily verified

$$^1\Pi_{f,g}^{f,g} = 1 \quad ^8\Pi_{f,g}^{f,g} = 8 \quad ^3\Pi_{f,g}^{f,g} = 3 \quad ^6\Pi_{f,g}^{f,g} = 6$$
$$^3P_{f,g,h}^{f,g,h} = 3 \quad ^6P_{f,g,h}^{f,g,h} = 6 \quad ^{15}P_{f,g,h}^{f,g,h} = 15$$

For the different amplitudes, the equations corresponding to the general form of eq.(2) are

$$
\begin{aligned}
-k^2 M_{a,c}^{b,d} = & -\frac{4\pi\alpha}{s^2} I_{a,c}^{b,d} + \frac{4\pi\alpha}{s}\left[\left(Q_{a,f}^{b,g}\left(-I_{g,c}^{f,d}\right) + I_{a,f}^{b,g} M_{g,c}^{f,d}\right)\rho\right.\\
& \left. + \left(M_{a,f}^{b,g} I_{g,c}^{f,d} + \left(-I_{a,f}^{b,g}\right)\bar{Q}_{g,c}^{f,d}\right)\bar{\rho} + \left(B_{a,fm}^{b,gl}\left(-J_{c,gl}^{d,fm}\right) + J_{a,fm}^{b,gl} \bar{B}_{c,gl}^{d,fm}\right)\gamma\right]\\
-k^2 Q_{a,c}^{b,d} = & \frac{4\pi\alpha}{s^2} I_{a,c}^{b,d} + \frac{4\pi\alpha}{s}\left[\left(Q_{a,f}^{b,g} I_{g,c}^{f,d} + I_{a,f}^{b,g} Q_{g,c}^{f,d}\right)\rho\right.\\
& \left. + \left(M_{a,f}^{b,g}\left(-I_{g,c}^{f,d}\right) + \left(-I_{a,f}^{b,g}\right)\bar{M}_{g,c}^{f,d}\right)\bar{\rho} + \left(B_{a,fm}^{b,gl} J_{c,gl}^{d,fm} + J_{a,fm}^{b,gl} B_{c,gl}^{d,fm}\right)\gamma\right]\\
-k^2 B_{a,cf}^{b,dg} = & \frac{4\pi\alpha}{s^2} J_{a,cf}^{b,dg} + \frac{4\pi\alpha}{s}\left[\left(Q_{a,l}^{b,m} J_{m,cf}^{l,dg} + I_{a,l}^{b,m} B_{m,cf}^{l,dg}\right)\rho\right.\\
& \left. + \left(M_{a,l}^{b,m}\left(-J_{m,cf}^{l,dg}\right) + \left(-I_{a,l}^{b,m}\right)\bar{B}_{m,cf}^{l,dg}\right)\bar{\rho}\right]
\end{aligned}
\tag{5}
$$

The coefficients ρ, $\bar{\rho}$, γ give the densities of quarks, antiquarks ans gluons; the equations for $\bar{Q}$ and $\bar{B}$ can be obtained by interchange $Q \Leftrightarrow \bar{Q}$, $\rho \Leftrightarrow \bar{\rho}$ and $B \Leftrightarrow -\bar{B}$ in the equations for Q and B. It is possible to give a graphical representation to eq.(5), see FIG.2, through the rules set in FIG.1; it is consistent with the colour structure to use lower indices for incoming quarks and outgoing antiquarks and upper indices for incoming antiquarks and outgoing quarks. Since the gluon colour structure is given in the spinorial version [App.A], the gluon is represented in the form of a $q\bar{q}$ pair, where up-arrow and down-arrow are used respectively for quarks and antiquarks.

2.2 Solution of the equations in the conjugate variables

We recall that the interaction term J is pure imaginary; in the last of eq.(5) it can be seen that B depends linearly on J, in the equations for M and Q, however, B and J appear always together and always multiplied by γ, so it is useful to perform the substitutions $J = -iJ'$; $B = -iB'$; in this way in the last equation we get a trivial factor $-i$, in the other two we get a change of sign in the terms that multiplies γ. In the whole set of eq (5) this amounts to eliminate the "prime" in J and B and to change the sign $\gamma \to -\gamma$. When we perform the decomposition

$$
\begin{aligned}
M = {}^1\Pi\, F_1 + {}^8\Pi\, F_8, \quad Q = {}^3\Pi\, F_3 + {}^6\Pi\, F_6 \quad \bar{Q} = {}^3\Pi\, \bar{F}_3 + {}^6\Pi\, \bar{F}_6\\
B = {}^3P\, B_3 + {}^6P\, B_6 + {}^{15}P\, B_{15} \quad \bar{B} = {}^3P\, \bar{B}_3 + {}^6P\, \bar{B}_6 + {}^{15}P\, \bar{B}_{15}
\end{aligned}
$$

we find contracting the interaction term J with the projectors P that the RHS of equations (5) for the $(qq, q\bar{q}, \bar{q}\bar{q})$ systems contain only the $I_{a,c}^{b,d}$ tensor; in fact in any case an octet is exchanged in the t-channel, in this way the relations

$$
F_8 = -\frac{1}{8}\, F_1, \quad F_6 = -\frac{1}{2}\, F_3, \quad \bar{F}_6 = -\frac{1}{2}\, \bar{F}_3
$$

hold as in the case without gluons and we get the equations:

$$
\begin{aligned}
k^2 F_1 - \frac{16\pi\alpha}{3 s^2} + \frac{4\pi\alpha}{s}\left[F_3\,\rho + \bar{F}_3\,\bar{\rho} + F_1\left(\frac{\rho+\bar{\rho}}{2}\right) + 2\left(\bar{B}_3 + \bar{B}_6 - B_3 - B_6\right)\gamma\right] = 0\\
k^2 F_3 - \frac{8\pi\alpha}{3 s^2} + \frac{4\pi\alpha}{s}\left[F_3\,\rho + F_1\left(\frac{\bar{\rho}}{2}\right) - 2\left(B_3 + B_6\right)\gamma\right] = 0\\
k^2 \bar{F}_3 - \frac{8\pi\alpha}{3 s^2} + \frac{4\pi\alpha}{s}\left[\bar{F}_3\,\bar{\rho} + F_1\left(\frac{\rho}{2}\right) + 2\left(\bar{B}_3 + \bar{B}_6\right)\gamma\right] = 0,
\end{aligned}
\tag{6}
$$

where B_{15} and $\bar{B}_{15}$ have been substituted by means of the following relations:

$$
B_{x,ac}^{x,bd} = \bar{B}_{x,ac}^{x,bd} = 0 \Rightarrow \quad 3 B_3 + 6 B_6 + 15 B_{15} = 0, \quad 3\bar{B}_3 + 6\bar{B}_6 + 15\bar{B}_{15} = 0.
$$

The system (6) yields immediately $F_1 = F_3 + \bar{F}_3$ and is reduced to a two-equation system:

$$F_3 \left[k^2 + \frac{4\pi\alpha}{s} \left(\rho + \frac{\bar{\rho}}{2} \right) \right] + \bar{F}_3 \left[\frac{2\pi\alpha\bar{\rho}}{s} \right] - \frac{8\pi\alpha}{3s^2} - \frac{8\pi\alpha}{s} \gamma (B_3 + B_6) = 0$$

$$F_3 \left[\frac{2\pi\alpha\rho}{2s} \right] + \bar{F}_3 \left[k^2 + \frac{4\pi\alpha}{s} \left(\bar{\rho} + \frac{\rho}{2} \right) \right] - \frac{8\pi\alpha}{3s^2} + \frac{8\pi\alpha}{s} \gamma (\bar{B}_3 + \bar{B}_6) = 0.$$

From the previous linear system it results that F_3 and $\bar{F}_3$ can be expressed in terms of $(B_3 + B_6)$ and $(\bar{B}_3 + \bar{B}_6)$, as follows:

$$F_3 = F_3^{(0)} + f_3^{(1)} (B_3 + B_6) + f_3^{(2)} (\bar{B}_3 + \bar{B}_6) \qquad\qquad \bar{F}_3 = \bar{F}_3^{(0)} + \bar{f}_3^{(1)} (B_3 + B_6) + \bar{f}_3^{(2)} (\bar{B}_3 + \bar{B}_6) \qquad (7)$$

where

$$F_3^{(0)} = \bar{F}_3^{(0)} = \frac{8\pi\alpha}{3s(k^2 s + 4\pi\alpha n)}$$

$$f_3^{(1)} = \frac{8\pi\alpha\gamma}{k^2 s + 4\pi\alpha n} + \frac{16\pi^2\alpha^2\gamma\bar{\rho}}{(k^2 s + 4\pi\alpha n)(k^2 s + 2\pi\alpha n)} \qquad f_3^{(2)} = \frac{16\pi^2\alpha^2\gamma\bar{\rho}}{(k^2 s + 4\pi\alpha n)(k^2 s + 2\pi\alpha n)}$$

$$\bar{f}_3^{(1)} = -\frac{16\pi^2\alpha^2\gamma\rho}{(k^2 s + 4\pi\alpha n)(k^2 s + 2\pi\alpha n)} \qquad \bar{f}_3^{(2)} = -\frac{8\pi\alpha\gamma}{k^2 s + 4\pi\alpha n} - \frac{16\pi^2\alpha^2\gamma\rho}{(k^2 s + 4\pi\alpha n)(k^2 s + 2\pi\alpha n)}$$

and $n = \rho + \bar{\rho}$. Furthermore we get

$$F_1 = 2 F_3^{(0)} + f_1^{(1)} (B_3 + B_6) + f_1^{(2)} (\bar{B}_3 + \bar{B}_6)$$

$$f_1^{(1)} = \frac{8\pi\alpha\gamma(k^2 s + 4\pi\alpha\bar{\rho})}{(k^2 s + 4\pi\alpha n)(k^2 s + 2\pi\alpha n)} \qquad\qquad f_1^{(2)} = -\frac{8\pi\alpha\gamma(k^2 s + 4\pi\alpha\rho)}{(k^2 s + 4\pi\alpha n)(k^2 s + 2\pi\alpha n)}$$

Projecting the equations satisfied by B and $\bar{B}$ with $^J P$ the following equations are obtained:

$$k^2 B_3 - \frac{6\pi\alpha}{s^2} + \frac{2\pi\alpha}{s} \left[\frac{9}{4} F_3 \rho + \frac{9}{8} F_1 \bar{\rho} + (B_3 \rho - \bar{B}_3 \bar{\rho}) \right] = 0 \qquad (8)$$

$$k^2 B_6 - \frac{2\pi\alpha}{s^2} + \frac{2\pi\alpha}{s} \left[\frac{3}{4} F_3 \rho + \frac{3}{8} F_1 \bar{\rho} + (B_6 \rho - \bar{B}_6 \bar{\rho}) \right] = 0 \qquad (9)$$

The equation for $\bar{B}_3$, $\bar{B}_6$ can obtained by the interchange $B_i \Leftrightarrow -\bar{B}_i$ for $i = 3, 6$, $\rho \Leftrightarrow \bar{\rho}$ and $F_3 \Leftrightarrow \bar{F}_3$ in the equations for B_3, B_6. After some tedious calculations, the following two equations for $(B_3 + B_6)$ and $(\bar{B}_3 + \bar{B}_6)$ variable are obtained:

$$\left[k^2 + \frac{2\pi\alpha}{s} \left(3 f_3^{(1)} \rho + \frac{3}{2} f_1^{(1)} \bar{\rho} + \rho \right) \right] (B_3 + B_6) + \left[\frac{2\pi\alpha}{s} \left(3 f_3^{(2)} \rho + \frac{3}{2} f_1^{(2)} \bar{\rho} - \bar{\rho} \right) \right] (\bar{B}_3 + \bar{B}_6)$$

$$- \frac{8\pi\alpha}{s^2} + \frac{6\pi\alpha}{s} F_3^{(0)} (\rho + \bar{\rho}) = 0$$

$$\left[\frac{2\pi\alpha}{s} \left(3 \bar{f}_3^{(1)} \bar{\rho} + \frac{3}{2} f_1^{(1)} \rho + \rho \right) \right] (B_3 + B_6) + \left[-k^2 + \frac{2\pi\alpha}{s} \left(3 \bar{f}_3^{(2)} \bar{\rho} + \frac{3}{2} f_1^{(2)} \rho - \bar{\rho} \right) \right] (\bar{B}_3 + \bar{B}_6)$$

$$- \frac{8\pi\alpha}{s^2} + \frac{6\pi\alpha}{2s} F_3^{(0)} (\rho + \bar{\rho}) = 0$$

Solving the previous linear system we have:

$$B_3 + B_6 = \frac{8\pi\alpha(k^2 s + 2\pi\alpha n)}{s(k^4 s^2 + 6 k^2 s\pi\alpha n + 48\pi^2\alpha^2\gamma n + 8\pi^2\alpha^2 n^2)}$$

$$\bar{B}_3 + \bar{B}_6 = -\frac{8\pi\alpha(k^2 s + 2\pi\alpha n)}{s(k^4 s^2 + 6 k^2 s\pi\alpha n + 48\pi^2\alpha^2\gamma n + 8\pi^2\alpha^2 n^2)} = -(B_3 + B_6)$$

Substituting the previous expressions in eq.(7) we have:

$$F_3 = \bar{F}_3 = F_3^{(0)} + \frac{32\,\pi^2\,\alpha^2\,\gamma}{s\,(k^4\,s^2 + 6\,k^2\,s\,\pi\,\alpha\,n + 48\,\pi^2\,\alpha^2\,\gamma\,n + 8\,\pi^2\,\alpha^2\,n^2)}$$
$$+ \frac{32\,\pi^2\,\alpha^2\,\gamma}{(k^4\,s^2 + 6\,k^2\,s\,\pi\,\alpha\,n + 48\,\pi^2\,\alpha^2\,\gamma\,n + 8\,\pi^2\,\alpha^2\,n^2)\,(k^2\,s + 4\,\pi\,\alpha\,n)} \tag{10}$$

From eq.(8,9,??,10) it can be easily verified that:

$$B_3 = -\bar{B}_3, \quad B_6 = -\bar{B}_6, \quad B_{15} = -\bar{B}_{15} \tag{11}$$

2.3 Correlations in real space

All the $q\bar{q}-$correlations may be expressed in term of F_3, so we evaluate its Laplace antitrasform with the following result:

$$\hat{G}_\beta(k^2) = \frac{2}{3\,n}\,\left[1 + \frac{6\,\gamma/n}{1 + 6\,\gamma/n}\right]\,\left[1 - e^{-\frac{3\,\pi\,n\,\alpha\,\beta}{k^2}}\left(\cos\left(\frac{3\,\pi\,n\,\alpha\,\beta\,y}{k^2}\right) - \frac{1 - 12\,\gamma/n}{3\,y\,(1 + 12\,\gamma/n)}\,\sin\left(\frac{3\,\pi\,n\,\alpha\,\beta\,y}{k^2}\right)\right)\right] \tag{12}$$

where $y = \frac{1}{3}\,\sqrt{48\,\gamma/n - 1}$. The inversion of the Fourier transform leads to an expression that is very little transparent, it is reported for completeness in [App.C]; the correlation function in real space has been computed numerically, by means of standard integration subroutines [7], starting form the following integral

$$G_\beta(r^2) = \frac{1}{2\,\pi\,r^2}\int_0^\infty dk\,k\,\sin(k\,r)\,\hat{G}_\beta(k^2); \tag{13}$$

In FIG.3 and FIG.4 the correlation function without and with gluons for different choices of temperature, at fixed coupling constant $\alpha = 0.2$ and initial quark density $b = 2\,fm^{-3}$, is shown. In both the cases it can be noted that the damping of the correlation, if not exponential, differs very little from that behavior; there is also a long distance r region (with $r \sim 8\,fm$ at $1/\beta = 350\,MeV$), where the correlation begin to oscillate, but the amplitude of these oscillations is very small, being present when the correlation has been already much damped. The previous plots give a good overall description of the behavior of the correlation function, but do not show immediately how these functions differ from a Yukawa shape $e^{-\mu\,r}/r$; in order to give a more complete description of this property in Fig. the expression $-\ln(r\,G_\beta(r))$ is plotted. If the Yukawa shape was exact, we would find a straight line; these plots confirm (obviously) that the gluons make the damping weaker, moreover the correlation function becomes slightly farther from a Yukawa shape. It can be seen that the effect associated to the gluon presence is to produce, at fixed temperature, a perceptible increase in the value of the correlation and consequently a displacement towards higher r values of the region where the correlation function begin to oscillate. For a detailed analysis of the values of energies, quark, anti-quark and gluon densities, we refer to the next section and [App.B].

3. CONCLUSIONS

The inclusion of the gluon in the effect of mutual screening in a $q\bar{q}-$plasma refines a previous analysis given in term of a pure quark-antiquark population, and it confirms the results. In particular the correlation length and the shape of the damping is still the same for qq and $q\bar{q}$, so that it must reflect in the same way in the meson and baryon production.

An item that requires some condideration is the comparison with other ways of dealing with the same phenomenon. The treatment given in term of strong coupling on the lattice is difficult to compare with [2, 3, 4] because it is very different since the beginning, more similar are the treaments in terms of thermal Green function as in [5]. In that case one extracts the contribution of the pole of the propagator in momentum space so that a precise Yukawa-like

decay of the correlation function is certainly produced. The treatment here presented is purely static, so it seems that it lacks of some effects that are present in the other treatment, does it contains also someting more? Perhaps what is more is seen looking at the structure of the coupled equation (5): the interaction of one particle takes place with other particles which are already correlated, so that at least at the level of two-fermion distribution a self consistent treatment of the correlations is performed.

It must be noted that the dependence of the shielding effect on the temperature has two origins: one is the kinematical effect that is present in every plasma-like system, the other is dynamical and typical of a relativistic system since it comes from the thermal production of particles, both gluons and $q\bar{q}$ pairs. The two processes have not the same role because initially there are more quarks than antiquarks, however, looking at the evolution of the chemical potential FIG.6, we see that there is an equivalent temperature, not extremely high, at which the effects of the initial condition become very small. One could also ask what the answer would be in the opposite limit, low temperature and high density, which might be realized inside a neutron star, but in this situation the Pauli principle would be very relevant and one should start from the consideration of a degenerate relativistic plasma, what is not attempted in this paper.

Appendix A: Quark-gluon interaction

As explained previously only the quark-gluon interaction corresponding to a gluon exchange in $t-$channel is considered. For this amplitude tha color structure is usually given in mixed form [6] as $f_{ABC}(T_C)^a_b$ where $A, B, C = 1 \ldots 8$ and $a, b = 1, 2, 3$. This form is not convenient here and will be transformed in a purely spinorial version. Using the fundamental commutator $[T_A, T_B] = if_{ABC}T_C$ we express the interaction as commutator; then defining the matrices with spinorial indices

$$(T^k_l)^a_b = \frac{1}{\sqrt{2}}\big[\delta^a_l \delta^k_b - \frac{1}{3}\delta^a_b \delta^k_l\big]$$

the form of the interactions used in this paper for qq and for qg is

$$I^{b,d}_{a,c} = \frac{1}{2}\left[\delta^d_a\delta^b_c - \frac{1}{3}\delta^b_a\delta^d_c\right] \qquad J^{b,dg}_{a,cf} = \frac{1}{2\,i}\left[\delta^d_a\delta^b_c\delta^g_f - \delta^g_a\delta^d_c\delta^b_f\right]$$

Note that the antisymmetry of J with respect to the exchange $(c, d) \leftrightarrow (f, g)$ together with the factor i ensures the Hermiticity of the interaction.

Appendix B: Densities of quarks, antiquarks and gluons

When we take care of the fact that there can be a thermal production of gluons we cannot ignore the concurrent production of quark-antiquark pairs. We investigate briefly the problem in this form. Assume the baryonic density, *i.e.* the quark density minus the antiquark density, as given and find the simultaneous production of gluons and of quark-antiquark pairs. The expression for the baryonic density is simple when the quark mass can be neglected, or gives at most a small correction, in this case for the density we get:

$$b = \frac{g_f}{2\,\pi^2}\int_0^\infty d\epsilon\left(\epsilon^2 - \frac{1}{2}m^2\right)\left[\frac{1}{e^{\beta\,(\epsilon-\mu)}+1} - \frac{1}{e^{\beta\,(\epsilon+\mu)}+1}\right] \tag{14}$$

In particular the antiquark density is given by

$$\bar{\rho} = \frac{g_f}{2\,\pi^2}\int_0^\infty \frac{d\epsilon\,(\epsilon^2 - \frac{1}{2}m^2)}{1+e^{\beta\,(\epsilon-\mu)}} = -\frac{g_f}{2\,\pi^2}\left[\frac{2}{\beta^3}\,\mathcal{L}_3\left(-e^{-\beta\,\mu}\right) + \frac{m^2}{2\beta}\,\ln(1+e^{-\beta\,\mu})\right] \tag{15}$$

where $\mathcal{L}_k(z) = \sum_{n=1}^\infty z^n/n^k$ and for the weight we get $g_f = 12$ resulting from a factor 2 for the spin, a factor 3 for the color, a factor 2 for the flavor because we consider only u and d quarks.

From eq (14) it follows:

$$b = \frac{g_f}{6\,\pi^2}\,\mu^3 + \frac{g_f}{6\,\beta^2}\,\mu - \frac{g_f}{4\,\pi^2}m^2\,\mu \tag{16}$$

From this relation we see the conditions qualitatively well known in which the mass term is negligible: either $m << 1/\beta$ or $m << \mu$, form now on we shall assume that at least one of these conditions is fulfilled and the mass term is dropped. Solving the equation for μ we find

$$\beta\,\mu = \frac{\left[\left(\frac{27\,\pi^2\,x}{g_f}\right)\left(1 + \sqrt{1 + \left(\frac{\pi\,g_f}{9\,\sqrt{3}\,x}\right)^2}\right)\right]^{\frac{2}{3}} - 3^{\frac{1}{3}}\,\pi^2}{3^{\frac{2}{3}}\left[\left(\frac{27\,\pi^2\,x}{g_f}\right)\left(1 + \sqrt{1 + \left(\frac{\pi\,g_f}{9\,\sqrt{3}\,x}\right)^2}\right)\right]^{\frac{1}{3}}} \tag{17}$$

with $x = b\,\beta^3$. In this way we get the actual expression of $\bar{\rho}$ in terms of b and β, the total density which appears in the previous formulae is given by $n = b + 2\bar{\rho}$.

It is now possible to give a semi-quantitative analysis of the different densities which are relevant for our problem: we start assigning a density b which we take equal to 2 fm^{-3}. In FIG.6, FIG.7 and FIG.8 the values of $\beta\mu$, n and the ratio γ/n as functions of $1/\beta$ are shown.

We have worked out three cases: the first is chosen so that the density of antiquarks is much less than b, we take $1/\beta = 150\,MeV$, this gives $\bar{\rho} \approx \frac{1}{20}\,b$ and for the corresponding gluon density $\gamma/n = 0.40$, in the second case we take $1/\beta = 300\,MeV$, this gives $\bar{\rho} \approx \frac{3}{2}\,b$ and $\gamma/n = 0.84$, in the third one $1/\beta = 450\,MeV$ is considered, which produces $\bar{\rho} \approx 6\,b$ and $\gamma/n = 0.88$, which is the "saturation" value for γ/n ratio. In these expression the density of gluons has be taken as the free-boson thermal density, which amounts to:

$$\frac{g_b}{2\pi^2}\frac{2}{\beta^3}\mathcal{L}_3(1) \tag{18}$$

Here $g_b = 16$ *i.e.* a factor 2 for the spin and a factor 8 for the color. Since we have $\beta\mu \to 0$ for $\beta \to 0$ it results that in that limit

$$\frac{\gamma}{n} = \frac{g_b}{2g_f}\frac{\mathcal{L}_3(1)}{-\mathcal{L}_3(-1)} = \frac{8}{9}$$

and this limit is numerically reached already at $1/\beta = 390$ MeV.

Appendix C: Analytical expression of the correlations

The inversion of the Fourier transform

$$G_\beta(r^2) = \frac{1}{(2\,\pi)^3}\int d^3k\,e^{i\,\mathbf{k}\cdot\mathbf{r}}\,\hat{G}_\beta(k^2)$$

implies a standard angular integration and then an integral over the radial coordinates that gives

$$G_\beta(r^2) = \frac{1}{2\,\pi\,r^2}\left[g_\beta^1\,_0F_2\left(;\frac{1}{2},2;\frac{3\,\pi\,n\,\alpha\,\beta\,(1-i\,y)}{4}\,r^2\right) + g_\beta^2\,_0F_2\left(;\frac{1}{2},2;\frac{3\,\pi\,n\,\alpha\,\beta\,(1+i\,y)}{4}\,r^2\right)\right.$$
$$\left. + g_\beta^3\,_0F_2\left(;\frac{3}{2},\frac{5}{2};\frac{3\,\pi\,n\,\alpha\,\beta\,(1-i\,y)}{4}\,r^2\right) + g_\beta^4\,_0F_2\left(;\frac{3}{2},\frac{5}{2};\frac{3\,\pi\,n\,\alpha\,\beta\,(1+i\,y)}{4}\,r^2\right)\right] \tag{19}$$

where the generalized hypergeometric functions are used

$$_pF_q\left(a_1,\ldots,a_p\,;\,b_1,\ldots,b_q\,;\,z\right) = \sum_{k=0}^{\infty}\frac{(a_1)_k\cdots(a_p)_k}{(b_1)_k\cdots(b_q)_k}\frac{z^k}{k!} \tag{20}$$

and the coefficient are

$$g_\beta^1 = \frac{\pi^2\,\alpha\,\beta\,(i+y)}{2\,y}\left(1+\frac{6\,\gamma/n}{1+6\,\gamma/n}\right)\left(\frac{1-12\,\gamma/n}{3\,(1+12\,\gamma/n)}-i\,y\right)$$

$$g_\beta^2 = \frac{\pi^2\,\alpha\,\beta\,(-i+y)}{2\,y}\left(1+\frac{6\,\gamma/n}{1+6\,\gamma/n}\right)\left(\frac{1-12\,\gamma/n}{3\,(1+12\,\gamma/n)}+i\,y\right)$$

$$g_\beta^3 = \frac{\sqrt{3\,\pi\,n\,(4\,\pi\,\alpha\,\beta)^3}\,(1-i\,y)^{\frac{3}{2}}\,r}{12\,(i\,y)}\left(1+\frac{6\,\gamma/n}{1+6\,\gamma/n}\right)\left(\frac{1-12\,\gamma/n}{3\,(1+12\,\gamma/n)}-i\,y\right)$$

$$g_\beta^4 = \frac{\sqrt{3\,\pi\,n\,(4\,\pi\,\alpha\,\beta)^3}\,(1+i\,y)^{\frac{3}{2}}\,r}{12\,(-i\,y)}\left(1+\frac{6\,\gamma/n}{1+6\,\gamma/n}\right)\left(\frac{1-12\,\gamma/n}{3\,(1+12\,\gamma/n)}+i\,y\right).$$

$$(21)$$

The function $G_\beta(r^2)$ is real because it is the sum of two terms with their complex conjugate. Note that for very low values of $1/\beta$, y could become imaginary, in this case all the addenda would be separately real.

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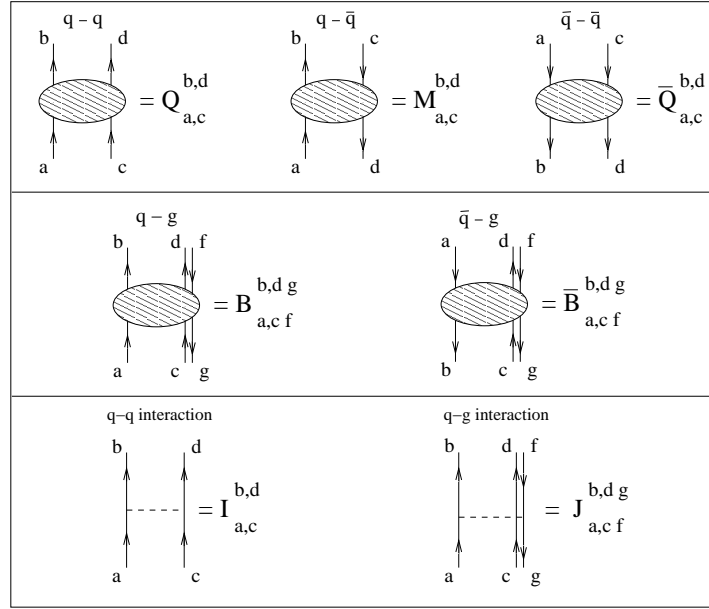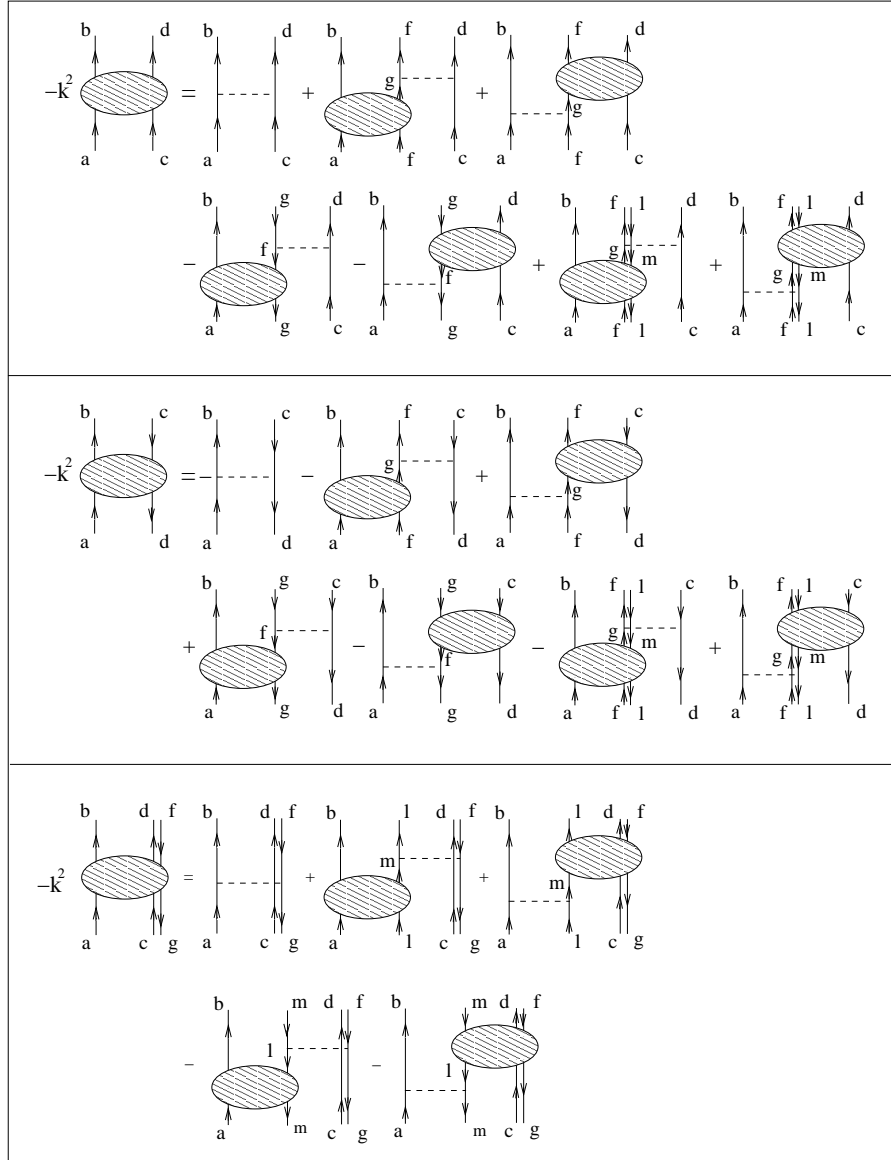[7] G. P. Lepage, J. Comp. Phys. **27**, 192 (1978).

FIG. 1: Rules for a graphical interpretation of eq.(5): up and down arrows indicate respectively quarks and antiquarks while the gluons are represented in the form of a $q\bar{q}$ pair(up-down arrow).
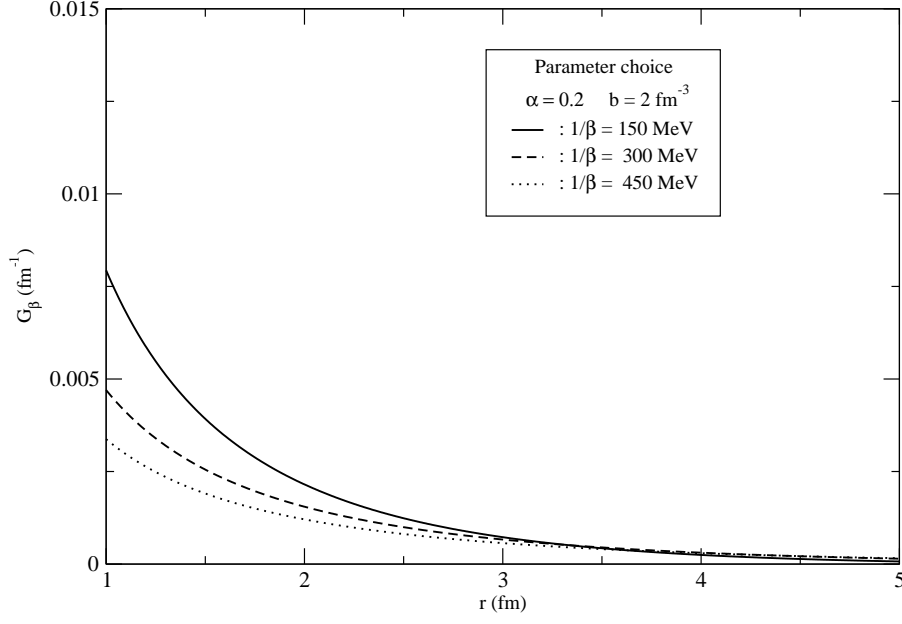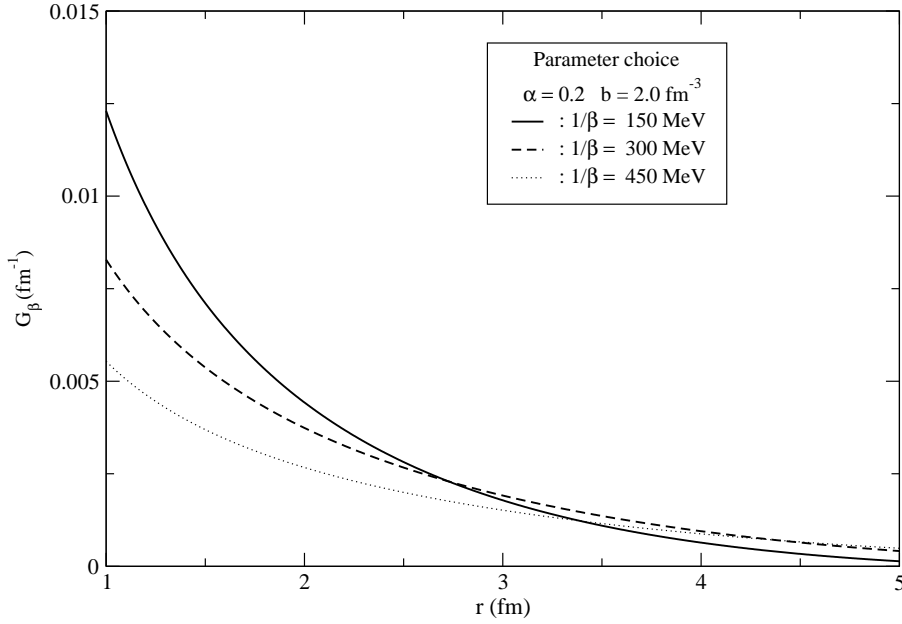
FIG. 3: Correlation function $G_\beta(r)$ in absence of gluons for different choices of temperatures, at fixed coupling constant $\alpha = 0.2$ and initial quark density $b = 2\,fm^{-3}$.
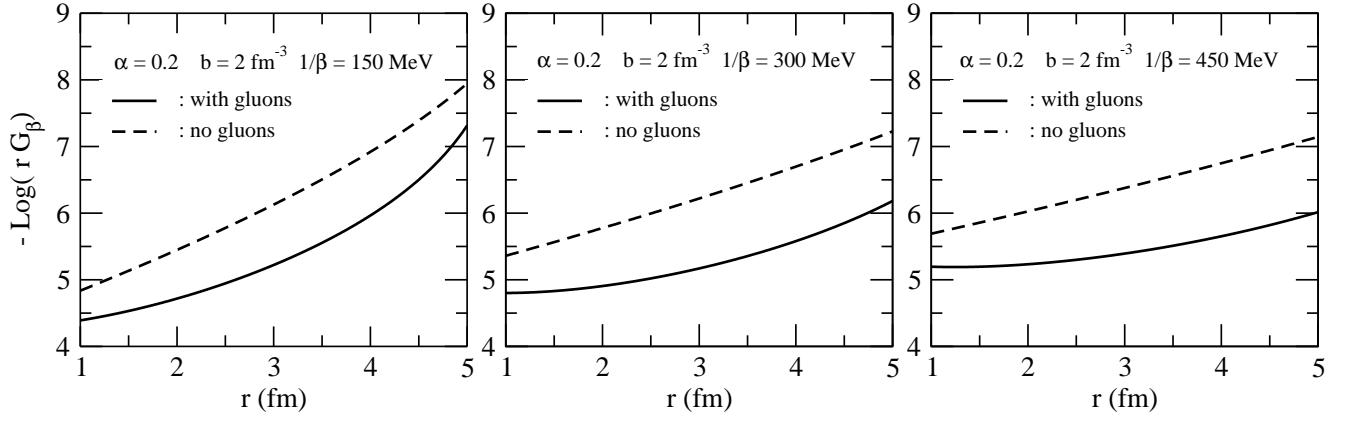

FIG. 4: Correlation function $G_\beta(r)$ in presence of gluons for different choices of temperatures, at fixed coupling constant $\alpha = 0.2$ and initial quark density $b = 2\,fm^{-3}$.

FIG. 5: Minus Logarithm of $r\,G_\beta(r)$ with (solid line) and without (dashed line) curve for different choices of temperature, coupling constant $\alpha = 0.2$ and initial quark density $b = 2\,fm^{-3}$.
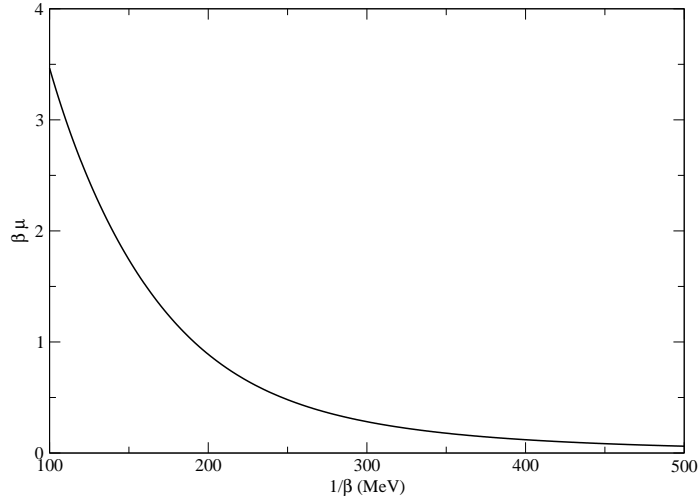

FIG. 6: $\beta\,\mu$ as a function of $1/\beta$ with initial quark density $b = 2\,fm^{-3}$; note that it is an adimensional quantity.

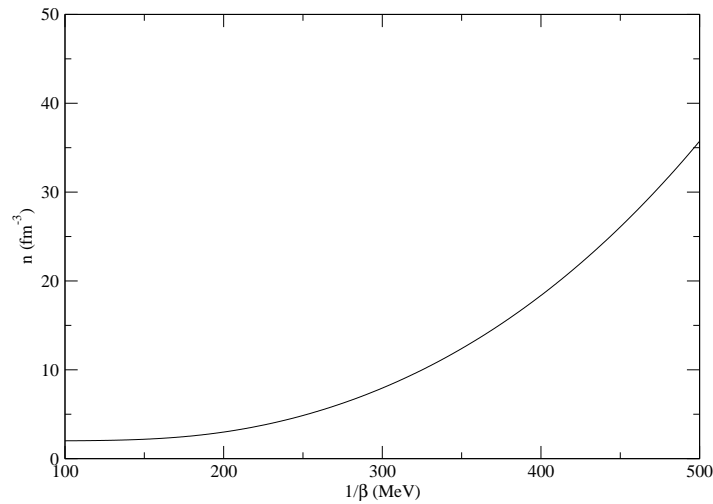FIG. 7: Fermion density as a function of $1/\beta$ with initial quark density is... $b = 2\,fm^{-3}$
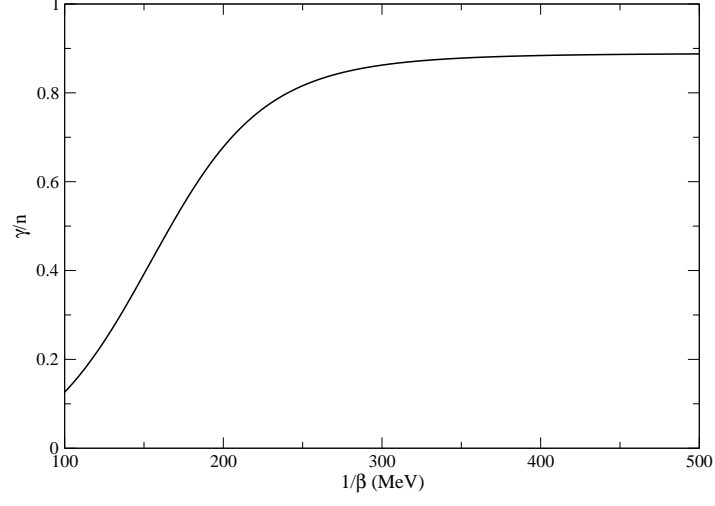
FIG. 8: γ/n ratio as a function of $1/\beta$ with initial quark density given by $b = 2\,fm^{-3}$.